NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2760

DERIVATION OF STABILITY CRITERIONS FOR BOX BEAMS

WITH LONGITUDINALLY STIFFENED COVERS

CONNECTED BY POSTS

By Paul Seide

Langley Aeronautical Laboratory
Langley Field, Va.

NACA

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SUMMARY

An investigation has been made of the elastic stability of an idealized box beam with longitudinally stiffened covers connected by posts and subjected to end moments and axial loads. Stability criterions which give the axial stiffness of the posts required to achieve desired stress values in the box-beam covers are derived.

INTRODUCTION

In a previous investigation of the use of posts as internal supporting elements in box beams (ref. 1), the results indicated that supports in the form of posts had possibilities for decreasing structural weight. The compression and tension covers of the idealized box beam considered in that investigation were unstiffened. Since unstiffened sheet is not the most efficient compression resistant element, the structural efficiency may be considerably improved when the sheet is stiffened by longitudinal stiffeners which, in turn, are supported by posts. An idealization of such a box beam is investigated in the present paper.

SYMBOLS

a_{mn} general coefficient in Fourier series for w

c integer defining location of post in X-direction, $1 \leqq c \leqq (M - 1)$

D plate flexural stiffness, $\dfrac{Et^3}{12(1 - \mu^2)}$

d   integer defining location of post in Y-direction,
                $1 \leqq d \leqq (N - 1)$

E   Young's modulus of elasticity of plate material

EI   flexural stiffness of longitudinal stiffeners

F   axial stiffness of posts, force per unit deflection

$g,h,i,j,m,$
n,r,s,c_1,c_2 } integers, also used as subscripts

k   plate-load coefficient, $l^2 N_x / \pi^2 D$

k_S stiffener-load coefficient, $l P_x / \pi^2 D$

L   longitudinal distance between posts

l transverse distance between posts

M   number of bays in X-direction

N   number of bays in Y-direction

N_x plate load per unit width

P_x stiffener load

p   integer defining number of buckles in Y-direction,
                $1 \leqq p \leqq (N - 1)$

q   integer defining number of buckles in X-direction,
                $1 \leqq q \leqq (M - 1)$

R   plate-flexural-stiffness ratio, D_T / D_C

S   post-axial-stiffness parameter, $F l^2 / \pi^4 D_C$

t   plate thickness

U   potential energy of box beam

U_S potential energy of longitudinal stiffeners

w   plate deflection in Z-direction

X,Y,Z   coordinate axes (see fig. 1)

x,y   distances along coordinate axes

β aspect ratio of plate between adjacent lines and rows of
                 posts, L/l

γ flexural-stiffness ratio, EI/lD

μ Poisson's ratio of plate material

Subscripts:

C   compression cover

T   tension cover

    Those parameters of the present paper that have the same symbols as in reference 1 have been redefined in terms of the transverse distance between rows of posts rather than the cover width because more natural parameters are thereby obtained.

DESCRIPTION OF STRUCTURE

    The structure of the present paper (fig. 1) differs from that of reference 1 in that longitudinal stiffeners of equal area and flexural stiffness are attached to the tension and compression covers and coincide with each row of posts. The stiffeners attached to the tension cover may differ from those attached to the compression cover. Each compression-cover stiffener is subjected to the same compressive load and each tension-cover stiffener, to the same tensile load. The stiffeners are considered to be concentrated at the middle surface of the cover to which they are attached and to have no torsional stiffness. As in reference 1, the covers consist of two simply supported flat rectangular plates of equal length and width which are connected in the interior by identical axially elastic posts in rectangular array. The plates may have unequal thicknesses and material properties and are subjected to uniform edge loadings, which may be of different intensities for the two plates. The connection between the plates and the posts is one of simple support. Both the longitudinal and transverse spacing of the posts are uniform; the two spacings, however, may differ. (See fig. 1.)

    The principal results of the investigation of this structure consist of elastic stability criterions which give the post axial stiffness required to achieve desired stress values in the covers of a given box beam at buckling.

DERIVATION OF STABILITY CRITERIONS

The method of derivation of the stability criterions of the present paper is similar to that of reference 1. Fourier series for the deflections of the compression and tension covers are substituted into the potential-energy expression for the structure; the potential-energy expression is then minimized with respect to the unknown coefficients in the deflection functions. A series of linear homogeneous equations is obtained from which stability criterions for the structure are determined. The algebraic manipulations that yield these criterions are somewhat more complex than those of reference 1 and are given in detail.

In order to obtain the potential energy of the present structure, it is necessary only to add the potential energy of the stiffeners to the potential energy of the structure considered in reference 1. The potential energy of the compression-cover stiffeners is given by

$$U_{S_C} = \sum_{d=1}^{N-1} \left\{ \frac{(EI)_C}{2} \int_0^{ML} \left[\left(\frac{\partial^2 w_C}{\partial x^2} \right)_{y=dl} \right]^2 dx - \frac{P_{x_C}}{2} \int_0^{ML} \left[\left(\frac{\partial w_C}{\partial x} \right)_{y=dl} \right]^2 dx \right\} \quad (1)$$

and of the tension-cover stiffeners, by

$$U_{S_T} = \sum_{d=1}^{N-1} \left\{ \frac{(EI)_T}{2} \int_0^{ML} \left[\left(\frac{\partial^2 w_T}{\partial x^2} \right)_{y=dl} \right]^2 dx + \frac{P_{x_T}}{2} \int_0^{ML} \left[\left(\frac{\partial w_T}{\partial x} \right)_{y=dl} \right]^2 dx \right\} \quad (2)$$

With w_C and w_T given by equations (1) and (2), respectively, of reference 1, these equations become

$$U_{S_C} = \frac{\pi^2}{4ML} \sum_{d=1}^{N-1} \sum_{m=1}^{\infty} m^2 \left[\frac{m^2 \pi^2}{M^2 L^2} (EI)_C - P_{x_C} \right] \left(\sum_{n=1}^{\infty} a_{mn_C} \sin \frac{n\pi d}{N} \right)^2 \quad (3)$$

and

$$U_{S_T} = \frac{\pi^2}{4ML} \sum_{d=1}^{N-1} \sum_{m=1}^{\infty} m^2 \left[\frac{m^2 \pi^2}{M^2 L^2} (EI)_T + P_{x_T} \right] \left(\sum_{n=1}^{\infty} a_{mn_T} \sin \frac{n\pi d}{N} \right)^2 \quad (4)$$

The potential energy of the structure is then the sum of equations (3) and (4) of the present paper and equation (6) of reference 1 and is given by the expression

$$
U = \frac{\pi^2}{8ML} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\left(m^2 + n^2 \frac{M^2}{N^2} \frac{L^2}{l^2} \right)^2 \frac{\pi^2}{M^2 L^2} D_C - m^2 N_{x_C} \right] a_{mn_C}^2 +
$$

$$
\frac{\pi^2}{8ML} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\left(m^2 + n^2 \frac{M^2}{N^2} \frac{L^2}{l^2} \right)^2 \frac{\pi^2}{M^2 L^2} D_T + m^2 N_{x_T} \right] a_{mn_T}^2 +
$$

$$
\frac{\pi^2}{4ML} \sum_{d=1}^{N-1} \sum_{m=1}^{\infty} m^2 \left[\frac{m^2 \pi^2}{M^2 L^2} (EI)_C - P_{x_C} \right] \left(\sum_{n=1}^{\infty} a_{mn_C} \sin \frac{n\pi d}{N} \right)^2 +
$$

$$
\frac{\pi^2}{4ML} \sum_{d=1}^{N-1} \sum_{m=1}^{\infty} m^2 \left[\frac{m^2 \pi^2}{M^2 L^2} (EI)_T + P_{x_T} \right] \left(\sum_{n=1}^{\infty} a_{mn_T} \sin \frac{n\pi d}{N} \right)^2 +
$$

$$
\frac{F}{2} \sum_{c=1}^{M-1} \sum_{d=1}^{N-1} \left[\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(a_{mn_C} - a_{mn_T} \right) \sin \frac{m\pi c}{M} \sin \frac{n\pi d}{N} \right]^2 \qquad (5)
$$

The buckling load is determined by the condition that the potential energy be a minimum; that is,

$$
\frac{\partial U}{\partial a_{ij_C}} = \frac{\partial U}{\partial a_{ij_T}} = 0 \qquad\qquad
\begin{array}{l}
(i = 1, 2, \ldots \infty) \\
(j = 1, 2, \ldots \infty)
\end{array}
$$

or

$$\left[\left(i^2 + j^2 \frac{M^2\beta^2}{N^2}\right)^2 - i^2 M^2\beta^2 k_C\right]a_{ij_C} +$$

$$\frac{2}{N} i^2\left(i^2\gamma_C - M^2\beta^2 k_{S_C}\right)\sum_{n=1}^{\infty} a_{in_C} \sum_{d=1}^{N-1} \sin\frac{n\pi d}{N}\sin\frac{j\pi d}{N} +$$

$$\frac{4M^3\beta^3}{N} S \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(a_{mn_C} - a_{mn_T})\left(\sum_{c=1}^{M-1}\sin\frac{m\pi c}{M}\sin\frac{i\pi c}{M}\right)\left(\sum_{d=1}^{N-1}\sin\frac{n\pi d}{N}\sin\frac{j\pi d}{N}\right) = 0$$

$$(6)$$

and

$$\left[\left(i^2 + j^2 \frac{M^2\beta^2}{N^2}\right)^2 + i^2 M^2\beta^2 k_T\right]a_{ij_T} +$$

$$\frac{2}{N} i^2\left(i^2\gamma_T + M^2\beta^2 k_{S_T}\right)\sum_{n=1}^{\infty} a_{in_T} \sum_{d=1}^{N-1} \sin\frac{n\pi d}{N}\sin\frac{j\pi d}{N} -$$

$$\frac{4M^3\beta^3}{N} \frac{S}{R} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(a_{mn_C} - a_{mn_T})\left(\sum_{c=1}^{M-1}\sin\frac{m\pi c}{M}\sin\frac{i\pi c}{M}\right)\left(\sum_{d=1}^{N-1}\sin\frac{n\pi d}{N}\sin\frac{j\pi d}{N}\right) = 0$$

$$(7)$$

$$(i = 1, 2, \ldots \infty)$$
$$(j = 1, 2, \ldots \infty)$$

Equations (6) and (7) may be separated into sets containing only particular coefficients a_{mn}. If j is of the form

$$j = c_1 N \qquad\qquad (c_1 = 1, 2, \ldots \infty)$$

that is, the covers buckle with longitudinal nodes through the stiffeners and posts, the summation $\sum_{d=1}^{N-1} \sin \frac{n\pi d}{N} \sin \frac{j\pi d}{N}$ is shown in reference 2 to be equal to zero. Then, particular equations for $j = c_1 N$ of equations (6) and (7) become

$$\left[\left(i^2 + c_1^2 M^2 \beta^2\right)^2 - i^2 M^2 \beta^2 k_C\right]\left(a_{ic_1 N}\right)_C = 0 \tag{8}$$

and

$$\left[\left(i^2 + c_1^2 M^2 \beta^2\right)^2 + i^2 M^2 \beta^2 k_T\right]\left(a_{ic_1 N}\right)_T = 0 \tag{9}$$

$$\begin{pmatrix} i = 1, \ 2, \ \ldots \ \infty \\ c_1 = 1, \ 2, \ \ldots \ \infty \end{pmatrix}$$

The stability criterions for this case are

$$k_C = \left(\frac{i}{M\beta} + c_1^2 \frac{M\beta}{i}\right)^2 \tag{10}$$

or

$$k_T = -\left(\frac{i}{M\beta} + c_1^2 \frac{M\beta}{i}\right)^2 \tag{11}$$

$$\begin{pmatrix} i = 1, \ 2, \ \ldots \ \infty \\ c_1 = 1, \ 2, \ \ldots \ \infty \end{pmatrix}$$

or both, in which case $k_T = -k_C$.

If i is of the form

$$i = c_2 M \qquad\qquad \left(c_2 = 1, \ 2, \ \ldots \ \infty\right)$$

and

$$j \neq c_1 N \qquad\qquad \left(c_1 = 1, \ 2, \ \ldots \ \infty\right)$$

that is, the covers buckle with transverse nodes through the posts, the
particular equations for $i = c_2 M$ of equations (6) and (7) reduce to

$$\left[\left(c_2^2 + j^2 \frac{\beta^2}{N^2}\right)^2 - c_2^2\beta^2 k_C\right]\left(a_{c_2 Mj}\right)_C +$$

$$\frac{2}{N} c_2^2\left(c_2^2\gamma_C - \beta^2 k_{S_C}\right)\sum_{n=1}^{\infty}\left(a_{c_2 Mn}\right)_C \sum_{d=1}^{N-1}\sin\frac{n\pi d}{N}\sin\frac{j\pi d}{N} = 0 \qquad (12)$$

and

$$\left[\left(c_2^2 + j^2 \frac{\beta^2}{N^2}\right)^2 + c_2^2\beta^2 k_T\right]\left(a_{c_2 Mj}\right)_T +$$

$$\frac{2}{N} c_2^2\left(c_2^2\gamma_T + \beta^2 k_{S_T}\right)\sum_{n=1}^{\infty}\left(a_{c_2 Mn}\right)_T \sum_{d=1}^{N-1}\sin\frac{n\pi d}{N}\sin\frac{j\pi d}{N} = 0 \qquad (13)$$

$$\left(c_2 = 1, 2, \ldots \infty\right)$$
$$\left(j \neq c_1 N; \quad c_1 = 1, 2, \ldots \infty\right)$$

These equations are similar in form to equations (A4) of reference 3 and
the stability criterions for this case can be written immediately as

$$\gamma_C - \frac{\beta^2 k_{S_C}}{c_2^2} + \cfrac{1}{\displaystyle\sum_{r=-\infty}^{\infty}\cfrac{1}{\left[1 + \left(2r + \frac{p}{N}\right)^2 \frac{\beta^2}{c_2^2}\right]^2 - \frac{\beta^2}{c_2^2}k_C}} = 0 \qquad (14)$$

$$\left(c_2 = 1, 2, \ldots \infty\right)$$

and

$$\gamma_T + \frac{\beta^2 k_{S_T}}{c_2^2} + \frac{1}{\displaystyle\sum_{r=-\infty}^{\infty} \frac{1}{\left[1 + \left(2r + \frac{p}{N}\right)^2 \frac{\beta^2}{c_2^2}\right]^2 + \frac{\beta^2}{c_2^2} k_T}} = 0 \qquad (15)$$

$$\left(c_2 = 1, 2, \ldots \infty\right)$$

In the cases considered thus far - buckling of the structure with longitudinal or transverse nodes through the posts - the posts were not compressed. The remaining equations of equations (6) and (7) determine the criterions for buckling of the box beam with compression of the posts. These equations can also be broken up into sets; each set of equations corresponds to a particular mode of buckling.

The values of i and j that have not been considered can be expressed as

$$i = 2sM + q$$

or

$$i = 2(s + 1)M - q$$

$$(s = 0, 1, 2, \ldots \infty)$$
$$(q = 1, 2, \ldots M - 1)$$

and

$$j = 2rN + p$$

or

$$j = 2(r + 1)N - p$$

$$(r = 0, 1, 2, \ldots \infty)$$
$$(p = 1, 2, \ldots N - 1)$$

When these values of i and j are substituted into the remaining equations of equations (6) and (7) and the summations $\displaystyle\sum_{c=1}^{M-1} \sin \frac{m\pi c}{M} \sin \frac{i\pi c}{M}$

and $\displaystyle\sum_{d=1}^{N-1} \sin\frac{n\pi d}{N}\sin\frac{j\pi d}{N}$ are evaluated by means of the table on page 8 of reference 2, the equations separate into eight groups. The remaining equations of equations (6) separate into

$$\left\{\left[\left(2s+\frac{q}{M}\right)^2+\left(2r+\frac{p}{N}\right)^2\beta^2\right]^2-\left(2s+\frac{q}{M}\right)^2 k_C\beta^2\right\}\left({}^a2sM+q,2rN+p\right)_C +$$

$$\left(2s+\frac{q}{M}\right)^2\left[\left(2s+\frac{q}{M}\right)^2\gamma_C - k_{S_C}\beta^2\right]\sum_{h=0}^{\infty}\left[\left({}^a2sM+q,2hN+p\right)_C -\right.$$

$$\left.\left({}^a2sM+q,2(h+1)N-p\right)_C\right] + \beta^3 SK_{pq} = 0 \qquad\qquad (16a)$$

$$\left(\left\{\left(2s+\frac{q}{M}\right)^2+\left[2(r+1)-\frac{p}{N}\right]^2\beta^2\right\}^2-\left(2s+\frac{q}{M}\right)^2 k_C\beta^2\right)\left({}^a2sM+q,2(r+1)N-p\right)_C -$$

$$\left(2s+\frac{q}{M}\right)^2\left[\left(2s+\frac{q}{M}\right)^2\gamma_C - k_{S_C}\beta^2\right]\sum_{h=0}^{\infty}\left[\left({}^a2sM+q,2hN+p\right)_C -\right.$$

$$\left.\left({}^a2sM+q,2(h+1)N-p\right)_C\right] - \beta^3 SK_{pq} = 0 \qquad\qquad (16b)$$

$$\left(\left\{\left[2(s+1)-\frac{q}{M}\right]^2+\left(2r+\frac{p}{N}\right)^2\beta^2\right\}^2-\left[2(s+1)-\frac{q}{M}\right]^2 k_C\beta^2\right)\left(a_{2(s+1)M-q,2rN+p}\right)_C +$$

$$\left[2(s+1)-\frac{q}{M}\right]^2\left\{\left[2(s+1)-\frac{q}{M}\right]^2\gamma_C-k_{S_C}\beta^2\right\}\sum_{h=0}^{\infty}\left[\left(a_{2(s+1)M-q,2hN+p}\right)_C -\right.$$

$$\left.\left(a_{2(s+1)M-q,2(h+1)N-p}\right)_C\right]-\beta^3 SK_{pq}=0 \qquad\qquad (16c)$$

$$\left(\left\{\left[2(s+1)-\frac{q}{M}\right]^2+\left[2(r+1)-\frac{p}{N}\right]^2\beta^2\right\}^2-\left[2(s+1)-\right.\right.$$

$$\left.\frac{q}{M}\right]^2 k_C\beta^2\right)\left(a_{2(s+1)M-q,2(r+1)N-p}\right)_C-\left[2(s+1)-\right.$$

$$\left.\frac{q}{M}\right]^2\left\{\left[2(s+1)-\frac{q}{M}\right]^2\gamma_C-k_{S_C}\beta^2\right\}\sum_{h=0}^{\infty}\left[\left(a_{2(s+1)M-q,2hN+p}\right)_C -\right.$$

$$\left.\left(a_{2(s+1)M-q,2(h+1)N-p}\right)_C\right]+\beta^3 SK_{pq}=0 \qquad\qquad (16d)$$

$$(s = 0, 1, 2, \ldots \infty)$$
$$(r = 0, 1, 2, \ldots \infty)$$
$$(p = 1, 2, \ldots N-1)$$
$$(q = 1, 2, \ldots M-1)$$

where

$$K_{pq} = \sum_{g=0}^{\infty} \sum_{h=0}^{\infty} \left[\left(a_{2gM+q,2hN+p} \right)_C - \left(a_{2gM+q,2(h+1)N-p} \right)_C - \right.$$

$$\left(a_{2(g+1)M-q,2hN+p} \right)_C + \left(a_{2(g+1)M-q,2(h+1)N-p} \right)_C - \left(a_{2gM+q,2hN+p} \right)_T +$$

$$\left. \left(a_{2gM+q,2(h+1)N-p} \right)_T + \left(a_{2(g+1)M-q,2hN+p} \right)_T - \left(a_{2(g+1)M-q,2(h+1)N-p} \right)_T \right]$$

and the remaining equations of equations (7) separate into

$$\left\{ \left[\left(2s + \frac{q}{M} \right)^2 + \left(2r + \frac{p}{N} \right)^2 \beta^2 \right]^2 + \left(2s + \frac{q}{M} \right)^2 k_T \beta^2 \right\} \left(a_{2sM+q,2rN+p} \right)_T +$$

$$\left(2s + \frac{q}{M} \right)^2 \left[\left(2s + \frac{q}{M} \right)^2 \gamma_T + k_{S_T} \beta^2 \right] \sum_{h=0}^{\infty} \left[\left(a_{2sM+q,2hN+p} \right)_T - \right.$$

$$\left. \left(a_{2sM+q,2(h+1)N-p} \right)_T \right] - \beta^3 \frac{S}{R} K_{pq} = 0 \qquad (17a)$$

$$\left(\left\{\left(2s + \frac{q}{M}\right)^2 + \left[2(r + 1) - \frac{p}{N}\right]^2 \beta^2\right\}^2 + \left(2s + \frac{q}{M}\right)^2 k_T \beta^2\right)\left(a_{2sM+q,\,2(r+1)N-p}\right)_T -$$

$$\left(2s + \frac{q}{M}\right)^2 \left[\left(2s + \frac{q}{M}\right)^2 \gamma_T + k_{S_T} \beta^2\right] \sum_{h=0}^{\infty} \left[\left(a_{2sM+q,\,2hN+p}\right)_T -\right.$$

$$\left.\left(a_{2sM+q,\,2(h+1)N-p}\right)_T\right] + \beta^3 \frac{S}{R} K_{pq} = 0 \tag{17b}$$

$$\left(\left\{\left[2(s + 1) - \frac{q}{M}\right]^2 + \left(2r + \frac{p}{N}\right)^2 \beta^2\right\}^2 + \right.$$

$$\left.\left[2(s + 1) - \frac{q}{M}\right]^2 k_T \beta^2\right)\left(a_{2(s+1)M-q,\,2rN+p}\right)_T + \left[2(s + 1) -\right.$$

$$\left.\frac{q}{M}\right]^2 \left\{\left[2(s + 1) - \frac{q}{M}\right]^2 \gamma_T + k_{S_T} \beta^2\right\} \sum_{h=0}^{\infty} \left[\left(a_{2(s+1)M-q,\,2hN+p}\right)_T -\right.$$

$$\left.\left(a_{2(s+1)M-q,\,2(h+1)N-p}\right)_T\right] + \beta^3 \frac{S}{R} K_{pq} = 0 \tag{17c}$$

$$\left(\left\{\left[2(s+1)-\frac{q}{M}\right]^2+\left[2(r+1)-\frac{p}{N}\right]^2\beta^2\right\}^2+\left[2(s+1)-\right.\right.$$

$$\left.\frac{q}{M}\right]^2 k_T\beta^2\right)\left(a_{2(s+1)M-q,2(r+1)N-p}\right)_T-\left[2(s+1)-\frac{q}{M}\right]^2\left\{\left[2(s+1)-\right.\right.$$

$$\left.\frac{q}{M}\right]^2\gamma_T+k_{S_T}\beta^2\right\}\sum_{h=0}^{\infty}\left[\left(a_{2(s+1)M-q,2hN+p}\right)_T-\right.$$

$$\left.\left(a_{2(s+1)M-q,2(h+1)N-p}\right)_T\right]-\beta^2\frac{S}{R}K_{pq}=0 \qquad\qquad (17d)$$

$$(s=0,\ 1,\ 2,\ \ldots\ \infty)$$
$$(r=0,\ 1,\ 2,\ \ldots\ \infty)$$
$$(p=1,\ 2,\ \ldots\ N-1)$$
$$(q=1,\ 2,\ \ldots\ M-1)$$

where K_{pq} is as previously defined. Each group of equations (16) and (17) corresponding to particular values of p and q is independent of the others inasmuch as only the deflection-function coefficients corresponding to these particular values of p and q appear in each group.

Consider equations (16a) and (16b) corresponding to some particular values of p and q. Divide each equation (16a) by the coefficient of $\left(a_{2sM+q,2rN+p}\right)_C$ and each equation (16b) by the coefficient of $\left(a_{2sM+q,2(r+1)N-p}\right)_C$. This process yields

$$\left(a_{2sM+q,\,2rN+p}\right)_C + \frac{\left(2s + \frac{q}{M}\right)^2\left[\left(2s + \frac{q}{M}\right)^2\gamma_C - \beta^2 k_{S_C}\right]}{\left[\left(2s + \frac{q}{M}\right)^2 + \left(2r + \frac{p}{N}\right)^2\beta^2\right]^2 - \left(2s + \frac{q}{M}\right)^2\beta^2 k_C} \sum_{h=0}^{\infty}\left[\left(a_{2sM+q,\,2hN+p}\right)_C - \right.$$

$$\left.\left(a_{2sM+q,\,2(h+1)N-p}\right)_C\right] + \frac{\beta^3 S}{\left[\left(2s + \frac{q}{M}\right)^2 + \left(2r + \frac{p}{N}\right)^2\beta^2\right]^2 - \left(2s + \frac{q}{M}\right)^2\beta^2 k_C} K_{pq} = 0 \qquad (18a)$$

and

$$\left(a_{2sM+q,\,2(r+1)N-p}\right)_C - \frac{\left(2s + \frac{q}{M}\right)^2\left[\left(2s + \frac{q}{M}\right)^2\gamma_C - \beta^2 k_{S_C}\right]}{\left\{\left(2s + \frac{q}{M}\right)^2 + \left[2(r+1) - \frac{p}{N}\right]^2\beta^2\right\}^2 - \left(2s + \frac{q}{M}\right)^2\beta^2 k_C} \sum_{h=0}^{\infty}\left[\left(a_{2sM+q,\,2hn+p}\right)_C - \right.$$

$$\left.\left(a_{2sM+q,\,2(h+1)N-p}\right)_C\right] - \frac{\beta^3 S}{\left\{\left(2s + \frac{q}{M}\right)^2 + \left[2(r+1) - \frac{p}{N}\right]^2\beta^2\right\}^2 - \left(2s + \frac{q}{M}\right)^2\beta^2 k_C} K_{pq} = 0 \qquad (18b)$$

$$(s = 0,\ 1,\ 2,\ \ldots\ \infty)$$
$$(r = 0,\ 1,\ 2,\ \ldots\ \infty)$$

Add all of the equations (18b) corresponding to a given value of s and subtract the resulting equation from the sum of all equations (18a) corresponding to a given value of s. The first terms of equations (18a) and (18b) then combine to yield

$$\sum_{r=0}^{\infty}\left[\left(a_{2sM+q,\,2rN+p}\right)_C - \left(a_{2sM+q,\,2(r+1)N-p}\right)_C\right]$$

which is the same as

$$\sum_{h=0}^{\infty}\left[\left(a_{2sM+q,\,2hN+p}\right)_C - \left(a_{2sM+q,\,2(h+1)N-p}\right)_C\right]$$

The resulting equation can then be written as

$$\left\{ 1 + \left(2s + \frac{q}{M}\right)^2 \left[\left(2s + \frac{q}{M}\right)^2 \gamma_C - \right.\right.$$

$$\beta^2 k_{S_C} \sum_{r \equiv \infty}^{\infty} \frac{1}{\left[\left(2s + \frac{q}{M}\right)^2 + \left(2r + \frac{p}{N}\right)^2 \beta^2\right]^2 \beta^2 - \left(2s + \frac{q}{M}\right)^2 \beta^2 k_C} \sum_{h=0}^{\infty} \left(a_{2sM+q,\,2hN+p}\right)_C -$$

$$\left.\left.\left(a_{2sM+q,\,2(h+1)N-p}\right)_C \right] + \beta^3 S K_{pq} \sum_{r=\infty}^{\infty} \frac{1}{\left[\left(2s + \frac{q}{M}\right)^2 + \left(2r + \frac{p}{N}\right)^2 \beta^2\right]^2 - \left(2s + \frac{q}{M}\right)^2 \beta^2 k_C} \right\} = 0 \qquad (19)$$

$$(s = 0, 1, 2, \ldots \infty)$$

Now divide each equation (19) by the coefficient of the first summation of deflection-function coefficients and add all of the resulting equations. Thus,

$$\sum_{s=0}^{\infty}\sum_{h=0}^{\infty}\left[\left(a_{2sM+q,\,2hN+p}\right)_C - \left(a_{2sM+q,\,2(h+1)N-p}\right)_C\right] +$$

$$\beta^3 S K_{pq}\sum_{s=0}^{\infty}\frac{1}{\left(2s+\dfrac{q}{M}\right)^2\left[\left(2s+\dfrac{q}{M}\right)^2\gamma_C - \beta^2 k_{S_C}\right]} + \sum_{r=-\infty}^{\infty}\frac{1}{\left[\left(2s+\dfrac{q}{M}\right)^2 + \left(2r+\dfrac{p}{N}\right)^2\beta^2\right]^2 - \left(2s+\dfrac{q}{M}\right)^2\beta^2 k_C} = 0$$

(20)

Similarly, equations (16c) and (16d) for particular values of p and q yield

$$\sum_{s=0}^{\infty}\sum_{h=0}^{\infty}\left[\left(a_{2(s+1)M-q,\,2hN+p}\right)_C - \left(a_{2(s+1)M-q,\,2(h+1)N-p}\right)_C\right] -$$

$$\beta^3 S K_{pq}\sum_{s=1}^{\infty}\frac{1}{\left(2s-\dfrac{q}{M}\right)^2\left[\left(2s-\dfrac{q}{M}\right)^2\gamma_C - \beta^2 k_{S_C}\right]} + \sum_{r=-\infty}^{\infty}\frac{1}{\left[\left(2s-\dfrac{q}{M}\right)^2 + \left(2r+\dfrac{p}{N}\right)^2\beta^2\right]^2 - \left(2s-\dfrac{q}{M}\right)^2\beta^2 k_C} = 0$$

(21)

equations (17a) and (17b) for particular values of p and q yield

$$\sum_{s=0}^{\infty}\sum_{h=0}^{\infty}\left[\left(a_{2sM+q,\,2hN+p}\right)_T - \left(a_{2sM+q,\,2(h+1)N-p}\right)_T\right]$$

$$-\frac{\beta^3 S}{R}K_{pq}\sum_{s=0}^{\infty}\left(2s+\frac{q}{M}\right)^2\left[\left(2s+\frac{q}{M}\right)^2\gamma_T+\beta^2 k_{S_T}\right]\cdot\frac{1}{\displaystyle\sum_{r=-\infty}^{\infty}\frac{1}{\left[\left(2s+\frac{q}{M}\right)^2+\left(2r+\frac{p}{N}\right)^2\beta^2\right]^2}+\left(2s+\frac{q}{M}\right)^2\beta^2 k_T}=0 \tag{22}$$

and equations (17c) and (17d) for particular values of p and q yield

$$\sum_{s=0}^{\infty}\sum_{h=0}^{\infty}\left[\left(a_{2(s+1)M-q,\,2hN+p}\right)_T - \left(a_{2(s+1)M-q,\,2(h+1)N-p}\right)_T\right]$$

$$-\frac{\beta^3 S}{R}K_{pq}\sum_{s=1}^{\infty}\left(2s-\frac{q}{M}\right)^2\left[\left(2s-\frac{q}{M}\right)^2\gamma_T+\beta^2 k_{S_T}\right]\cdot\frac{1}{\displaystyle\sum_{r=-\infty}^{\infty}\frac{1}{\left[\left(2s-\frac{q}{M}\right)^2+\left(2r+\frac{p}{N}\right)^2\beta^2\right]^2}+\left(2s-\frac{q}{M}\right)^2\beta^2 k_T}=0 \tag{23}$$

Finally, add equations (21) and (22) and subtract the resulting equation from the sum of equations (20) and (23). The first terms of equations (20) to (23) combine to give K_{pq}. The stability criterion for buckling of the structure with q buckles in the X-direction (longitudinal) and p buckles in the Y-direction (transverse) can then be obtained as

$$\frac{1}{\beta^3 S} + \sum_{s=-\infty}^{s=\infty} \frac{1}{\left(2s+\frac{q}{M}\right)^2 \left[\left(2s+\frac{q}{M}\right)^2 \gamma_C - \beta^2 k_{S_C}\right]} + \sum_{r=-\infty}^{r=\infty} \frac{1}{\left[\left(2s+\frac{q}{M}\right)^2 + \left(2r+\frac{p}{N}\right)^2 \beta^2\right]^2 - \left(2s+\frac{q}{M}\right)^2 \beta^2 k_C} +$$

$$\frac{1}{R} \sum_{s=-\infty}^{s=\infty} \frac{1}{\left(2s+\frac{q}{M}\right)^2 \left[\left(2s+\frac{q}{M}\right)^2 \gamma_T + \beta^2 k_{S_T}\right]} + \sum_{r=-\infty}^{r=\infty} \frac{1}{\left[\left(2s+\frac{q}{M}\right)^2 + \left(2r+\frac{p}{N}\right)^2 \beta^2\right]^2 + \left(2s+\frac{q}{M}\right)^2 \beta^2 k_T} = 0 \qquad (24)$$

Since repetition of the method of solution for the equations corresponding to other values of p and q would give the same stability criterion as equation (24), p and q in this equation can take on the values

$$p = 1, 2, \ldots N - 1$$

$$q = 1, 2, \ldots M - 1$$

Equations (10), (11), (14), (15), and (24) constitute the complete set of stability criterions. If

$$\gamma_C = \gamma_T = k_{S_C} = k_{S_T} = 0$$

the stability criterions reduce to those of reference 1, after allowances have been made for differences in the parameters used.

CONCLUDING REMARKS

Criterions for the elastic stability under end moment and axial load of an idealized box beam with longitudinally stiffened covers connected by posts have been derived. These stability criterions give the axial stiffness of the posts required to achieve desired stress values in the box-beam covers.

The parameters used in the present paper are defined in terms of the transverse spacing of the rows of posts rather than the cover width because more natural parameters are thereby obtained.

Langley Aeronautical Laboratory
  National Advisory Committee for Aeronautics
    Langley Field, Va., June 3, 1952

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3. Seide, Paul, and Stein, Manuel: Compressive Buckling of Simply Supported Plates With Longitudinal Stiffeners. NACA TN 1825, 1949.
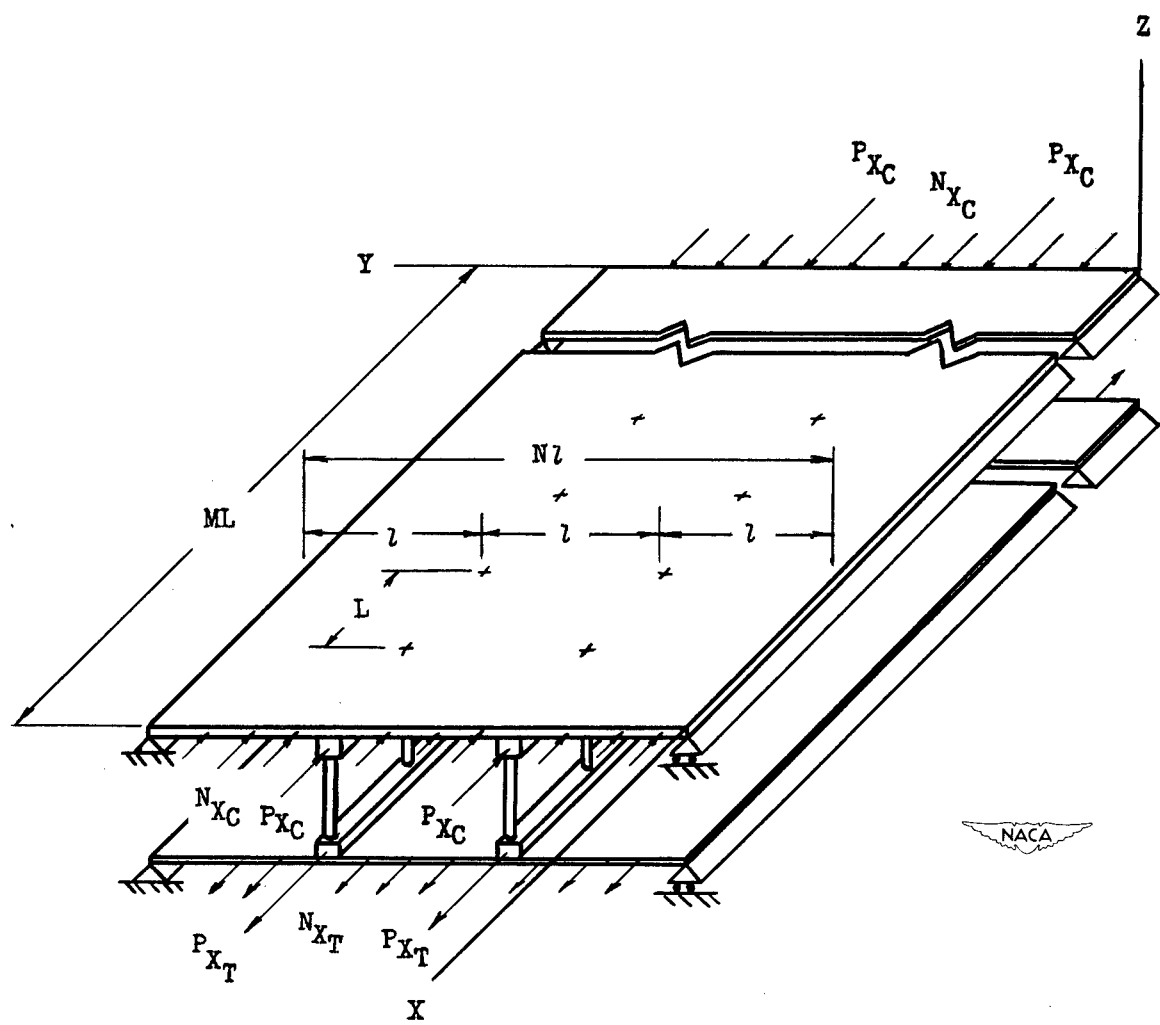
Figure 1.- Idealization of a box beam with longitudinally stiffened covers connected by posts.

NACA TN 2760
National Advisory Committee for Aeronautics.
DERIVATION OF STABILITY CRITERIONS FOR
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August 1952. 21p. diagr. (NACA TN 2760)

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stiffened covers connected by posts and subjected to
end moments and axial loads. Stability criterions
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achieve desired stress values in the box-beam covers
are derived.

Copies obtainable from NACA, Washington

1. Plates, Flat - Stiffened
   (4.3.3.1.2)
2. Beams, Box (4.3.4.1)
3. Loads and Stresses,
   Structural - Tension
   (4.3.7.1)
4. Loads and Stresses,
   Structural - Compres-
   sion (4.3.7.2)
5. Loads and Stresses,
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